small singular set. Using this the following result on the differentiability of the stable norm S was presented

Theorem 1 (F. Auer, V. Bangert)**.** *For $h \in H_{n-1}(M, \mathbb{R})$ let $V(h)$ denote the smallest subspace of $H_{n-1}(M, \mathbb{R})$ that contains h and is generated by integer classes. Then $S|_{V(h)}$ is differentiable at h.*

In the recent thesis by H. Junginger-Gestrich (Freiburg) it is shown that for $M = \mathbb{T}^n$ this result is optimal, in the sense that for a large open set of metrics on $\mathbb{T}^n$ the stable norm is only differentiable in the directions given in the theorem.

Projective dynamics of a classical particle or a multiparticle system

Alain Albouy

(1) *If two Riemannian metrics on a manifold have the same pre-geodesics (i.e. unparametrized geodesics), then their geodesic flows are integrable.* This statement needs some more hypotheses but it is however quite striking. It was discussed in 1998 by Matveev and Topalov [9], and independently by Tabachnikov (see [11]), and applied to the geodesic flow on the n-dimensional ellipsoid. The discussion is based on the paper [6] by Levi-Civita.

(2) *If a Newton system, i.e. a system of the form $\ddot{q} = f(q)$, with $q \in \Omega \subset \mathbb{R}^n$, $f : \Omega \to \mathbb{R}^n$ being a smooth function, possesses two quadratic first integrals then it is integrable.* Again the statement is astonishing, and it requires some technical hypotheses (some seem generic but are not often satisfied in the examples while others seem quite unlikely to happen but do happen). It is due to Lundmark's thesis in 1999 (see e.g. [7]).

(3) *The geodesic flow on the ellipsoid is after a change of time the Neumann problem on this ellipsoid seen as a sphere* (i.e. choosing the Euclidean structure that makes the ellipsoid a sphere). More precisely it is a energy level of Neumann's problem. This is due to Knoerrer [5].

(4) *Appell's central projection sends Neumann's problem onto a Newton system*, if we define this projection as follows. The particle moves on a hypersphere under a quadratic potential (Neumann). We choose a hyperplane not passing through the center of the sphere, and project on it the particle motion using the central projection from the center of the sphere. Finally we apply Appell's change of time [2].

Whatever be the hyperplane, the projected system possesses two quadratic first integrals satisfying Lundmark's hypotheses. Thus Statements (3) (4) and (2) give an elegant way to reach the main example of Statement (1): the geodesic flow on the ellipsoid.

(4') If this hyperplane is parallel to a coordinate hyperplane for the coordinates diagonalizing the quadratic potential, the projected system is naturally Hamiltonian. It is number -1 in the bi-infinite Jacobi family of separable potentials, defined by Rauch-Wojciechowski [12]. This integrable Newton system was also noticed by Appell [3].

Levi-Civita was trying to extend Appell's surprising transformation from the projectively flat to the curved framework. Levi-Civita paper was used by dozens of authors while Appell was being forgotten (we don't know any mention of his transformation in the period 1952–2002). If Appell's transformation was quite unpopular, it is maybe because it is not symplectic, it does not respect the time parameter, and the function called Energy before transformation has nothing to do with a possible energy after transformation. However, we developed in [1] very elementary and concrete consequences of Appell's remark. As an example, we give the simplest way to find the Hamiltonian of the projected system (4').

Neumann's Hamiltonian on the sphere $\|q\| = 1$, where $q \in \mathbb{R}^{n+1}$, with potential $U(q) = \langle aq, q\rangle/2$ is

$$H = \frac{1}{2}\big(\|q\|^2\|p\|^2 - \langle q, p\rangle^2\big) + \frac{1}{2}\langle aq, q\rangle.$$

In a base where the symmetric matrix a is diagonal with diagonal $(a_0, \dots, a_n)$ one of the Uhlenbeck-Devaney first integrals is

$$F_0 = \sum_{i=1}^{n} \frac{(q_0p_i - q_ip_0)^2}{a_0 - a_i} + q_0^2.$$

Projective dynamics (a possible name for considerations around Appell's central projection) teaches us that there is a unique homogeneous form for each of these first integrals. We find it using [1]:

$$\tilde{H} = \frac{1}{2}\big(\|q\|^2\|p\|^2 - \langle q, p\rangle^2\big) + \frac{1}{2}\frac{\langle aq, q\rangle}{\|q\|^2}. \qquad \tilde{F}_0 = \sum_{i=1}^{n} \frac{(q_0p_i - q_ip_0)^2}{a_0 - a_i} + \frac{q_0^2}{\|q\|^2}.$$

Note that we did not need to change the velocity dependent term of these first integrals. We were lucky: in general the homogeneization of this term requires a computation. For example, if we had written above H in the simpler way $H = (\|p\|^2 + \langle aq, q\rangle)/2$, the deduction of $\tilde{H}$ would require a computation. We took the expressions of H and F_0 in Moser's papers (e.g. [8]) but only readers who are familiar with Moser's *constrained Hamiltonian systems* can understand why he expressed H in this complicated way. Moser happened to write the homogeneous form of the velocity dependent term, and his motivations seem unrelated to projective dynamics.

The operation opposite to homogeneization is restriction. If we restrict $\tilde{H}$ and $\tilde{F}_0$ to the sphere $\|q\| = 1$ we find H and F_0. If we restrict them to $q_0 = 1$, together with the associated tangent condition $p_0 = 0$, we find:

$$\bar{H} = \frac{1}{2}\big((1 + q_1^2 + \dots + q_n^2)(p_1^2 + \dots + p_n^2) - (q_1p_1 + \cdots)^2\big) + \frac{a_0 + a_1q_1^2 + \dots + a_nq_n^2}{2(1 + q_1^2 + \dots + q_n^2)}$$

$$\bar{F}_0 = \sum_{i=1}^{n} \frac{p_i^2}{a_0 - a_i} + \frac{1}{1 + q_1^2 + \dots + q_n^2}.$$

In these expressions we make $p_i = \dot{q}_i$ and they become the first integrals of some Newton system. There is a unique Newton system having $\bar{F}_0$ as a first integral and this system is the Hamiltonian system associated with the Hamiltonian $\bar{F}_0/2$ expressed in the momenta $P_i = p_i/(a_0 - a_i)$. This is Appell's or Rauch-Wojciechowski's system. We see that H was the Hamiltonian, and F_0 just a quadratic first integral, and now $\bar{H}$ is just a quadratic first integral while $\bar{F}_0/2$ is the Hamiltonian. In the terminology of Magri's school the system is quasi-bi-Hamiltonian ("quasi" because time is changed, see [10]).

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Global Fixed Points for Group Actions and Morita's Theorem

John Franks

This talk concerned the existence of global fixed points for certain smooth group actions on surfaces.

Theorem 1 (Franks, Handel, Parwani [1]). *Let $\mathcal{G}$ be an abelian subgroup of $\mathit{Diff}^1_0(\mathbb{R}^2)$ with the property that there is a compact $\mathcal{G}$ invariant subset of $\mathbb{R}^2$. Then there is a point $x \in \mathbb{R}^2$ such that $g(x) = x$ for all g in $\mathcal{G}$.*