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Action minimizing solutions of the Newtonian n-body problem

Alain Chenciner

Astronomie et Systèmes Dynamiques, IMCCE, UMR 8028 du CNRS,
77, avenue Denfert-Rochereau, 75014 Paris, France
and
Département de Mathématiques, Université Paris VII-Denis Diderot
16, rue Clisson, 75013 Paris, France

As early as 1896, Henri Poincaré [P] proposed to find periodic solutions of the planar 3-body problem by minimizing the Lagrangian action over a well-chosen class of loops in the configuration space for the problem. He was well aware of the main obstacle to this approach : due to the weakness of Newton's potential, the action of a solution stays finite through collision and a minimizer may well be composed of segments of true solutions connecting collision configurations. This is indeed quite often the case when one tries to minimize over the homology or homotopy class of the loop and this forced Poincaré to replace Newton's potential by a "strong force" one.

The minimization method has recently been given a new impetus by the discovery that the replacement of topological constraints by symmetry ones often leads to collision-free minimizers. A typical example of such a symmetry constraint is permitted by the invariance of the Lagrangian under permutation of equal masses. Ask for the invariance of the loop under the action of Z/nZ whose generator cyclically permutes the bodies after one n-th of the period. Minimization over such a class leads to the "choreographies", whose name, given by their main producer Carles Simó, comes from the beautiful figures they display on the screen in computer experiments. Referring to my survey article [C] for a bibliography and an overview of the few new solutions for which a proof is available (Chenciner-Venturelli's Hip-Hop for 4 bodies in space, Chenciner-Montgomery's Eight for 3 bodies in the plane and Chen's Parallelogram solution for 4 bodies in the plane), I shall concentrate in the lecture on a proof of the

Theorem. *Let us consider two configurations A and B (eventually with collisions) of the n-body problem in R^3 or R^2 with arbitrary masses and a real number $T > 0$. A minimizer of the action among paths joining A to B in time T has no collision and hence is a true solution of Newton's equations.*

Based on a remarkable idea of Christian Marchal [M] which solves the case of isolated collisions with a fixed limit configuration before and after the collision, a complete proof of this theorem uses ideas of Richard Montgomery, Susanna Terracini and Andrea Venturelli to cope with the two main technical problems : possible accumulation of collision times and possible existence of continua of similitude classes of central configurations.

Useless in the case of topological constraints, this theorem turns out to be very pertinent in the case of symmetry constraints because it reduces the possibility of collisions in a minimizer to the two instants which are the boundaries of a segment of the loop serving as a fundamental domain for the action of the symmetry group. Some consequences will be given. In particular we will construct Marchal's $P12$ family which connects the Eight to the Lagrange equilateral relative equilibrium solution through spatial choreographies in a rotating frame.

[P] Poincaré H. *Sur les solutions périodiques et le principe de moindre action, C.R.A.S. t. 123, pp. 915-918, 1896*

[C] Chenciner A. *Action minimizing periodic solutions of the n-body problem, "Celestial Mechanics, dedicated to Donald Saari for his 60th Birthday", A. Chenciner, R. Cushman, C. Robinson, Z.J. Xia editors, Contemporary Mathematics 292 (2002)*

[M] Marchal C. *How the method of minimization of action avoids singularities, Celmec III, Roma (June 2001)*