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On the long term evolution of the spin of the Earth

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Abstract. Laskar and Robutel (1993) have globally analyzed the stability of the planetary obliquities in a conservative framework. Here the same model is extended by adding dissipative effects in the Earth–Moon system: the body tides and the friction between the core and the mantle. Some constraints on the poorly known coefficients of dissipation are determined with the help of paleogeological observations. One consequence is that the scenario proposed by Williams (1993) for the past history of the Earth's obliquity seems unlikely. A synthesis of 500 numerical integrations of the Earth–Moon system with orbital perturbations for the next 5 Gyr is presented. It is shown that the time scale of the dissipative effects is long enough to induce an adiabatic–like evolution of the obliquity which is driven in the chaotic zone within 1.5 to 4.5 Gyr. A statistical study of possible evolutions conducted with a tidal dissipation coefficient Δt of 600 seconds demonstrated that 68.4% of the trajectories attained an obliquity larger than 81 degrees, with a maximum of 89.5 degrees.

Key words: Earth – chaos – instabilities – celestial mechanics

1. Introduction

The recent works (Laskar *et al.*, 1993a-b) and (Laskar and Robutel, 1993) emphasized the sensitivity of the obliquity ϵ of a planet to the planetary perturbations. Indeed, secular resonances between the precession motion of the rotation axis of a planet and the slow secular motion of its orbit due to planetary perturbations can result in large chaotic variations of its obliquity. In the case of the Earth, the presence of the Moon changes the Earth's precessing frequency by a large amount, and thus keeps it in a stable region, far from the large chaotic zone which results from secular resonances overlap (Laskar *et al.*, 1993b). But due to tidal dissipation, the Moon is slowly receding from the Earth and the Earth's rotation is slowing down. Ultimately, the Earth will reach the large chaotic zone due to planetary perturbations, and its obliquity will no longer be stable. The object of the present work is to provide a quantitative description of the long term evolution of the Earth's obliquity, in the future, but also in the past.

The evolution of the Earth–Moon system is far from being a new subject. Evidence of the loss of the Earth's angular momentum has long been observed through paleogeological clocks (for a review, see Williams, 1989), while the present deceleration of the lunar mean motion can be directly measured by Lunar Laser Ranging (Dickey *et al.*, 1994) with great precision. Nevertheless, accurate quantitative estimates of the length of the day over the age of the Earth are still lacking, and it is still a difficult question to know the precise origins of these evolutions. Apart from the early work of (Darwin, 1880), the major works on the past history of the Earth-Moon system are due to MacDonald (1964), Goldreich (1966) and Mignard (1979, 1980, 1981). They found the same trends in the variations of the Earth's spin and the lunar orbit due to the effects of the tides raised on the Earth by the Sun and the Moon. Adding the planetary perturbations to the Earth's orbit, Touma and Wisdom (1994) recently confirmed those past variations, the rates of which nevertheless remain uncertain because the coefficient of tidal dissipation is not well known. On the other hand, none of these studies have taken into account the action of the friction between the core and the mantle of the Earth as has been done in studies of Venus' obliquity (see for example Goldreich and Peale, 1970, Lago and Cazenave, 1979, Dobrovolskis, 1980, Yoder, 1995). However, as is pointed out by Williams (1993), this effect could be of great importance, providing the spin with an obliquity decreasing with the time. The controversy about the efficiency of the core–mantle friction arises from the fact that the possible effective viscosities of the outer core cover a very large range of values (Lumb and Aldridge, 1991), and in the scenario presented by Williams (1993) for the past evolution of the Earth's obliquity, a very high value of this viscosity is assumed in order to obtain a past obliquity of the Earth reaching 70 degrees one billion years ago.

On the other hand, the future of the evolution of the Earth's obliquity has already been explored by Ward (1982) which showed, using a simple model with isolated resonances, that the precession frequency is expected to cross planetary secular resonances in the future, which could allow the obliquity to increase up to 60 degrees. The reality is much more severe, as, from the extended work (Laskar *et al.*1993b) and (Laskar and Robutel, 1993), we know that as the Moon recedes from the Earth, as soon as the obliquity reaches the first important

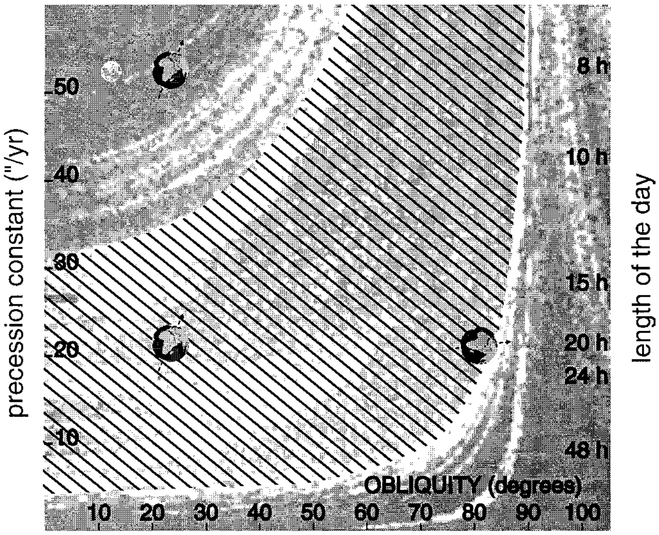

Fig. 1. The zone of large scale chaotic behavior for the obliquity of the Earth for a wide range of precession constant α (in arcseconds per year). The non-hatched region corresponds to the stable orbits, where the variations of the obliquity are moderate, while the hatched zone is the chaotic zone. The chaotic behavior is estimated by the diffusion rate of the precession frequency measured for each initial condition (ϵ, α) via numerical frequency map analysis over 36 Myr. In the large chaotic zones, the chaotic diffusion will occur on horizontal lines (α is fixed), and the obliquity of the planet can explore horizontally all the hatched zone. With the Moon, one can consider the present situation of the Earth to be represented approximately by the point with coordinates $\epsilon = 23.44°, \alpha = 54.93\,''/yr$, which is in the middle of a large zone of regular motion. Without the Moon, for spin periods ranging from about 12h to 48h, the obliquity of the Earth would undergo very large chaotic variations ranging from nearly 0° to about 85°. This figure summarizes the results of about 250 000 numerical integrations of the Earth's obliquity variations under perturbations due to the whole solar system for various initial conditions over 36 Myr. (Laskar and Robutel, 1993, Laskar *et al.*1993b).

planetary secular resonance, it will enter a very large chaotic zone, with the possibility of attaining very high obliquities up to nearly 90 degrees.

Indeed, using Laskar's method of frequency map analysis over more than 250 000 numerical integrations of the Earth's obliquity for various values of the precession constant, it was possible to obtain a clear picture of the global dynamics of the Earth's obliquity (Laskar and Robutel, 1993) (Fig.1). Each point of the graph represents one value of the couple (obliquity, precession constant), the precession constant being a quantity proportional to the speed of rotation (see formula (1) in Sect. 2). One dot in the non-hatched zone corresponds to a stable position, where the obliquity suffers only small (nearly quasiperiodic) variations around its mean value, whereas one point in the hatched zone corresponds to a chaotic behavior so that the hatched area delimits a region of resonances overlap where the Earth can wander horizontally. The present Earth is located in a stable region (ϵ = 23.44 degrees, α = 54.93 arcsec/year), and the present variations of the obliquity are limited to ±1.3 degrees around its mean value (Laskar *et al.*, 1993a).

Fig. 1 can be considered as a snapshot of the dynamics of the Earth's obliquity, constructed in a conservative framework, over a relatively short time scale on which the dissipation due to tidal interaction or core-mantle coupling is not yet visible. However, this picture already allows to forecast the future and past evolutions of the Earth's obliquity on much longer time scales, of several billions of years, when the various dissipative effects can no longer be neglected. Indeed, the consequence of this dissipation is to slow down the rotation of the Earth, so that an initial point of the graph is slowly brought down to lower values of the precession constant. This suggests that the Earth's spin has smoothly evolved since the formation or capture of the Moon. Our aim in the present work is to give a precise view of the future evolution of the Earth's obliquity, and more specifically, to describe quantitatively its path in the chaotic zone.

The main limitation on a precise evolution of the Earth's rotational state is as much the crudeness and uncertainty of dissipative models as it is the values of their parameters, such as the amplitude of the tidal dissipation, and even more, the viscosity of the outer core. The choice of the dissipative model does not seem to be fundamental here, essentially because the Earth's speed of rotation is not subject to large changes: different models would not lead to very different variations, especially when compared to the ones produced by planetary perturbations. Besides, in order to overcome difficulties arising from the uncertainty of the parameters, we will rely on the geological observations of the length of the day (Williams, 1989). This will allow us to obtain a set of plausible values which correspond to these observations, and to fix the time scale of the evolution along the way.

Section 2 is devoted to determine the averaged equations of secular rotational dynamics with planetary perturbations. In Sect. 3 we present the chosen models for estimating the additional dissipative contributions to these equations, while some limits for the coefficients of dissipation will be obtained in Sect. 4. Then we discuss in Sect. 5 the history of the Earth's obliquity proposed by Williams (1993). Finally, we present in Sect. 6 the results of a set of 500 numerical simulations of the future evolution of the Earth for the next 5 Gyr, using Laskar's method of integration of the solar system (Laskar, 1988, 1994a), and starting with very close initial conditions. Indeed, the chaotic nature of the motion prevents us from computing a single orbit, and only a statistical approach becomes meaningful for this problem. Before concluding, we discuss some alternatives to the results in relation to the coefficients of dissipation.

2. Averaged equations for the precession of the Earth with planetary perturbations

The equations of precession of the Earth are derived from a Hamiltonian function H which is the sum of the kinetic energy and of the potential energy U_p of the torque exerted by the Sun and the Moon on the equatorial bulge of the Earth.

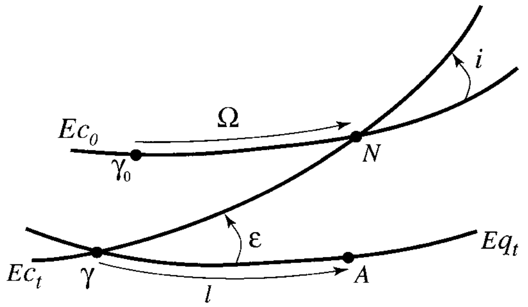

Fig. 2. Reference frames for the definition of precession. Eq_t and Ec_t are the mean equator and ecliptic of the date with equinox γ. Ec_0 is the fixed J2000 ecliptic, with equinox γ_0. i is the inclination of the ecliptic Ec_t on Ec_0. The general precession in longitude, ψ is defined by $\psi = \gamma N + N\gamma_0$, where N is the ascending node of the ecliptic of date on the fixed ecliptic. ϵ is the obliquity, and $\ell = \gamma A$ the hour angle of the equinox of date γ.

We suppose here that the Earth is an homogeneous rigid body with moments of inertia $A < B < C$ and we assume that its spin axis is also the principal axis of inertia. It is convenient here to use canonical Andoyer's action variables (L, X) and their conjugate angles $(\ell, -\psi)$ (Andoyer, 1923, Kinoshita, 1977) (Fig. 2). $L = C\omega$ is the modulus of rotational angular momentum of the Earth with rotation angular velocity ω; $X = L\cos\epsilon$, is the projection of the angular momentum on the normal to the ecliptic, at obliquity ϵ; ℓ is the hour angle between the equinox of the date γ and a fixed point of the equator; $-\psi$ the opposite of the general precession angle (see Fig. 2).

Let $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ be a reference frame fixed with respect to the Earth, $(\mathbf{I}, \mathbf{J}, \mathbf{K})$ a reference frame linked to the orbital plane of the perturbing body (Sun or Moon) around the Earth (see Fig. 2), and $\mathscr{R}$ the rotation such that $(\mathbf{i}, \mathbf{j}, \mathbf{k}) = \mathscr{R}(\mathbf{I}, \mathbf{J}, \mathbf{K})$.

According to Tisserand (1891) or Smart (1953) the potential energy of the torque exerted by a perturbing body $\mathscr{P}$ of mass $\mathscr{M}$ at distance r from the Earth ($\mathscr{E}$), limited to its largest component is

$$U_p = -\frac{G\mathscr{M}}{r}\left[\frac{A+B+C-3\mathscr{T}}{2r^2} + O\left(\left(\frac{R}{r}\right)^3\right)\right]$$

where R is the Earth's radius. $\mathscr{T}$ denotes the Earth's moment of inertia around the radius vector $\mathbf{r} = \overrightarrow{\mathscr{E}P}$, and is given by

$$\mathscr{T} = A + \frac{1}{r^2}(B-A)(\mathbf{r}\cdot\mathscr{R}(\mathbf{J}))^2 + \frac{1}{r^2}(C-A)(\mathbf{r}\cdot\mathscr{R}(\mathbf{K}))^2 .$$

The motion of $\mathscr{P}$ around the Earth is determined by its elliptical orbital elements defined with respect to the fixed ecliptic E_{c_0}, with reference direction toward the fixed equinox γ_0. Let us denote $\bar{\omega}$ its argument of perigee, v its true anomaly, $w_d = \Omega + \bar{\omega} + \psi + v$ the true longitude of date, where Ω is the longitude of ascending node of the apparent orbit of $\mathscr{P}$ on E_{c_0} if $\mathscr{P}$ is the Sun, and on E_{c_t} if it is the Moon.

We first build the precession equations due to the perturbation of the Sun only. When $\mathscr{P}$ is the Sun (subscript $\odot$), we have:

$$\mathbf{r} = \mathbf{r}_\odot = r_\odot\big(\cos(\bar{\omega}_\odot + v_\odot)\mathbf{I} + \sin(\bar{\omega}_\odot + v_\odot)\mathbf{J}\big)$$

and the transformation from the equatorial frame to the ecliptic one $(\mathbf{I}, \mathbf{J}, \mathbf{K})$ is:

$$\mathscr{R} = \mathscr{R}_3(-\psi - \Omega_\odot)\,\mathscr{R}_1(\epsilon)\,\mathscr{R}_3(\ell)\ ,$$

where the rotations $\mathscr{R}_1$ and $\mathscr{R}_3$ are defined as

$$\mathscr{R}_1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \quad ;$$

$$\mathscr{R}_3(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

Hence

$$\frac{1}{r_\odot^2}\big(\mathbf{r}_\odot\cdot\mathscr{R}(\mathbf{J})\big)^2 = \tfrac{1}{4}\big[(1-\cos 2\ell\cos 2w_{d\odot})(1+\cos^2\epsilon) + (\cos 2w_{d\odot} - \cos 2\ell)\sin^2\epsilon - 2\cos\epsilon\,\sin 2\ell\sin 2w_{d\odot}\big] .$$

$$\frac{1}{r_\odot^2}\big(\mathbf{r}_\odot\cdot\mathscr{R}(\mathbf{K})\big)^2 = \tfrac{1}{2}\sin^2\epsilon\,(1-\cos 2w_{d\odot})$$

We retain only the contribution of terms with no spherical symmetry, which gives with Andoyer's variables

$$U_{p_\odot} = \frac{3Gm_\odot}{4r_\odot^3}\left\{(C-A)\left(1-\frac{X^2}{L^2}\right)(1-\cos 2w_{d\odot}) + \frac{B-A}{2}\left[\left(1-\cos 2\ell\,\cos 2w_{d\odot}\right)\left(1+\frac{X^2}{L^2}\right) + \left(\cos 2w_{d\odot} - \cos 2\ell\right)\left(1-\frac{X^2}{L^2}\right) - 2\frac{X}{L}\sin 2\ell\sin 2w_{d\odot}\right]\right\}.$$

Let $M_\odot$ be the mean anomaly of the Sun. The fast angles ℓ and $w_{d\odot}$ are removed by taking the average

$$\overline{U}_{p_\odot} = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{1}{2\pi}\int_0^{2\pi} U_{p_\odot} d\ell\right) dM_\odot,$$

unless a spin–orbit resonance occurs, i.e. when the angle $\ell - \frac{k}{2}M_\odot$ ($k \in \mathbf{Z}$) is librating (Peale, 1969). This leads to the following expression for the averaged potential due to the Sun:

$$\overline{U}_{p_\odot} = -\frac{3Gm_\odot}{4a_\odot^3}\,C(1-e_\odot^2)^{-3/2}E_d\frac{X^2}{L^2}$$

where $E_d = (2C - A - B)/(2C)$ is called the dynamical ellipticity. The contribution of the Moon (subscript M) to the Hamiltonian follows the same procedure with

$$\mathbf{r} = r_M\mathscr{R}_M(\mathbf{I})$$

where $\mathscr{R}_M = \mathscr{R}_3(\Omega_M)\mathscr{R}_1(i_M)\mathscr{R}_3(\bar{\omega}_M + v_M)$. Assuming a constant rate for the precession of the orbit of the Moon (node and perihelion), one can also average the subsequent U_{pM} on ℓ, Ω_M and M_M, which gives:

$$\overline{U}_{pM} = -\frac{3Gm_M}{4{a_M}^3} C(1-{e_M}^2)^{-3/2}\big(1-\tfrac{3}{2}\sin^2 i_M\big)E_d\frac{X^2}{L^2}$$

where i_M is the inclination of the lunar orbit on the ecliptic. The full averaged Hamiltonian function of the described motion is then obtained by adding the rotational kinetic energy $T = \frac{1}{2}C\omega^2 = L^2/2C$, which gives

$$H = \frac{L^2}{2C} - \frac{\alpha}{2}\frac{X^2}{L}$$

where α is the "precession constant":

$$\alpha = \frac{3G}{2\omega}\left[\frac{m_\odot}{(a_\odot\sqrt{1-{e_\odot}^2})^3} + \frac{m_M}{(a_M\sqrt{1-{e_M}^2})^3}(1-\tfrac{3}{2}\sin^2 i_M)\right]E_d \qquad (1)$$

For a fast rotating planet like the Earth, E_d can be considered as proportional to ω^2 ; this correspond to the hydrostatic equilibrium (see for example Lambeck, 1980). In this approximation, α is proportional to ω.

Now, when considering the perturbations of the other planets, the ecliptic E_{c_t} is not an inertial plane any more and the kinetic energy E of its driving has to be added. We refer here to Kinoshita (1977).

Let $(L^\star, X^\star, \ell^\star, \psi^\star)$ be Andoyer's variables relative to the fixed ecliptic E_{c_0}, and (L, X, ℓ, ψ) the variables relative to the ecliptic E_{c_t}. Then (see for example Kovalevsky, 1963), if K is the Hamiltonian of the system, function of the variables (L, X, ℓ, ψ) relative to the moving E_{c_t}, and F the Hamiltonian written with the variables $(L^\star, X^\star, \ell^\star, \psi^\star)$ relative to E_{c_0}, the transformation

$$\mathscr{T} : (L^\star, X^\star; \ell^\star, -\psi^\star) \longmapsto (L, X; \ell, -\psi)$$

is canonical if, and only if, there exists a total differential form dW such that

$$Ld\ell + Xd(-\psi) - L^\star d\ell^\star - X^\star d(-\psi^\star) - (K-F)dt = dW. \qquad (2)$$

The expression $K - F$ is the searched energy E.

In the previous section, H was the Hamiltonian F written with the new variables (L, X, ℓ, ψ). Then, the new Hamiltonian $K = E + H$ can be obtain by identifying E and dW in the Eq. (2). Thanks to Danjon (1959), one can establish the following relation:

$$\cos(\ell^\star - \ell) = \cos(-\psi - \Omega)\,\cos(-\psi^\star - \Omega) + \sin(-\psi - \Omega)\,\sin(-\psi^\star - \Omega)\,\cos i$$

where i is the inclination of E_{c_t} on the fixed plane E_{c_0}. Then, if the obliquity ϵ is oriented from the rotation axis $\mathbf{k}$ to the orbit normal $\mathbf{K}$, we have

$$d(\ell^\star - \ell) = \cos\epsilon\; d(-\psi - \Omega) - \cos\epsilon^\star\, d(-\psi^\star - \Omega) - \sin(-\psi - \Omega)\,\sin\epsilon\, di$$

where $\epsilon^\star$ is the obliquity relative to E_{c_0}. As $\cos\epsilon^\star = \cos\epsilon\,\cos i + \sin\epsilon\sin i\cos(-\psi - \Omega)$ (Danjon, 1959) and $L = L^\star$, we finally obtain

$$Ld\ell - Xd\psi - L^\star d\ell^\star + X^\star d\psi^\star - \big[(X(1-\cos i) - L\sin\epsilon\sin i\cos(-\psi-\Omega)\big]\,d\Omega - L\sin\epsilon\sin(-\psi-\Omega)\,di = 0$$

which leads to

$$dW = 0$$

and

$$E = \big[(X(1-\cos i) - L\sin\epsilon\,\sin i\cos(\Omega+\psi)\big]\,\frac{d\Omega}{dt} - L\sin\epsilon\,\sin(\Omega+\psi)\,\frac{di}{dt}$$

or

$$E = 2\mathscr{C}(t)X - L\sqrt{1-\tfrac{X^2}{L^2}}\,\big(\mathscr{A}(t)\sin\psi + \mathscr{B}(t)\cos\psi\big)$$

with

$$\begin{cases} \mathscr{A}(t) = \dfrac{2}{\sqrt{1-p^2-q^2}}\big[\dot{q} + p(q\dot{p} - p\dot{q})\big] \\ \mathscr{B}(t) = \dfrac{2}{\sqrt{1-p^2-q^2}}\big[\dot{p} - q(q\dot{p} - p\dot{q})\big] \\ \mathscr{C}(t) = q\dot{p} - p\dot{q} \end{cases}$$

and where $q = \sin(i/2)\cos\Omega$ and $p = \sin(i/2)\sin\Omega$. The canonical equations $dX/dt = \partial K/\partial\psi$ and $d\psi/dt = -\partial K/\partial X$ then give the precession equations on the form (Laskar, 1986, Laskar *et al.*, 1993a-b):

$$\begin{cases} \dfrac{dX}{dt} = L\sqrt{1-\tfrac{X^2}{L^2}}\,\big(\mathscr{B}(t)\sin\psi - \mathscr{A}(t)\cos\psi\big) \\ \dfrac{d\psi}{dt} = \dfrac{\alpha X}{L} - \dfrac{X}{L\sqrt{1-\tfrac{X^2}{L^2}}}\big(\mathscr{A}(t)\sin\psi + \mathscr{B}(t)\cos\psi\big) - 2\mathscr{C}(t) \end{cases}$$

As was already done by Laskar *et al.*(1993b) and Laskar and Robutel (1993), since the contribution of the planetary perturbations to $\dot{\psi}$ is singular for $\epsilon = 0$, we use for numerical integrations, instead of (X, ψ), the complex variable

$$\chi = (1-\cos\epsilon)e^{i\psi}$$

which moves the singularity to $\epsilon = 180°$.

$\mathscr{A}$, $\mathscr{B}$ and $\mathscr{C}$ depend on fundamental frequencies of the solar system and they are implicitly given by the integration of the planetary motions. In this context, α is obviously not a constant: $e_\odot$ is also given by the integration of the solar system; ω, a_M, e_M and i_M have to be determined as functions of time because of the dissipation.

3. Contributions of dissipative effects

Now we give estimations for the averaged contributions of the dissipation to dL/dt and dX/dt due to tides and core-mantle interaction. For a review of the major features concerning the evolution of the Earth–Moon system, one can refer to the book edited by Marsden and Cameron (1966). It helped us to delimit what was important for this study.

3.1. The body tides

One specific problem of modeling the dissipative tides on Earth is that they have two different origins: friction in the mantle and friction within shallow seas. The simplest way to overcome difficulties and large formulae is to link the *global* dissipation to one physical quantity. In particular, this leads to consider here the Earth, once again, as homogeneous.

The specific dissipation function Q (Munk and MacDonald, 1960) is often used. It is defined as the inverse of the ratio $\Delta E/(2\pi E_0)$ where ΔE is the energy dissipated during one period of tidal stress and E_0 the maximum of energy stored during the same period. MacDonald (1964) showed that

$$Q^{-1} \simeq \tan\delta \tag{3}$$

where δ is the phase lag of the deformation due to the stress.

Q is rather considered as a constant (see for example Kaula, 1964, Goldreich and Peale, 1967, Goldreich and Soter, 1969, Gold and Soter, 1969), what implies in particular that δ is independent of the speed of rotation. This point remains subject to controversy, especially for long time scales. Some others studies have also considered δ dependent of the tidal frequency: (Goldreich and Peale, 1966, 1970), (Lambeck, 1979), (Lago and Cazenave, 1979), (Dobrovolskis, 1980). Most of them use Fourier expansions of the tidal potential (Kaula, 1964) in which an arbitrary tidal phase lag has to be defined for each argument, and the way these phase lags are related to the frequency is not always clear. Moreover, relation (3) itself is subject to uncertainty as was pointed out by Zschau (1978).

As Touma and Wisdom (1994), we prefer here the simpler and more intuitive approach of Mignard (1979) where the torque resulting of tidal friction is proportional to the time lag Δt that the deformation takes to reach the equilibrium. This time lag is supposed to be constant, and the angle between the direction of the tide–raising body and the direction of the axis of minimal inertia (i.e. the direction of the high tide), which is carried out of the former by the rotation of the Earth, is proportional to the speed of rotation. Such a model is called "viscous", and corresponds to the case for which $1/Q$ is proportional to the tidal frequency.

Theories on tidal effects are generally based on the following assertion mainly due to Love at the beginning of the century (see Lambeck, 1988): the tidal potential due to the deformation induced by the differential gravitational attraction of a perturbing body (the Sun or the Moon) at $\mathbf{r}^*$ from the Earth's center O holds, at any point P on its surface:

$$V(\mathbf{r}^*, \mathbf{R}) = \sum_{i\geq 2} k_i V_i(\mathbf{r}^*, \mathbf{R}),$$

where $\mathbf{R} = \overrightarrow{OP}$ of which the modulus is the planetary radius R, where k_i the i^{th} Love number and V_i the i^{th} spherical harmonic. As was done for the computation of the potential of precession, we restrict ourselves to the first term of the expansion, what seems to be sufficient for estimating secular variations. Hence

$$V(\mathbf{r}^*, \mathbf{R}) = k_2 V_2(\mathbf{r}^*, \mathbf{R}) = -\frac{Gm^* k_2}{2r^{*5}}\left(3(\mathbf{R}\cdot\mathbf{r}^*)^2 - R^2 r^{*2}\right) \tag{4}$$

The potential V at any point outside the Earth of distance r' is the solution of the "Dirichlet's first boundary–value problem" (see Lambeck, 1988), so to say that it satisfies Laplace's equation

$$\nabla^2 V = 0$$

and its value $V(\mathbf{r}^*, \mathbf{R})$ on the boundary of the domain $\{r' > R\}$ is known and given by Eq. (4). The unique solution to this problem is the function

$$V(\mathbf{r}^*, \mathbf{r}') = k_2 \left(\frac{R}{r'}\right)^5 V_2(\mathbf{r}^*, \mathbf{r}').$$

Here $\mathbf{r}^*$ stands for the perturbing body and $\mathbf{r}'$ for the interacting body at time t. $\mathbf{r}^*$ would also be defined at time t if the Earth were perfectly elastic. But this is not the case since, due to internal friction, the deformation permanently takes a time Δt to reach the equilibrium; the attribute $(^*)$ referring to the perturbing body also means that the value is taken at time $t - \Delta t$.

Assuming Δt small compared to the diurnal period, Mignard gives the following approximation for the perturbing body:

$$\mathbf{r}^* = \mathbf{r}(t-\Delta t) + \overrightarrow{\omega}\Delta t \times \mathbf{r} = \mathbf{r} + (\overrightarrow{\omega}\times\mathbf{r} - \mathbf{v})\Delta t$$

where $\mathbf{v}$ is the orbital velocity, $\overrightarrow{\omega}$ the rotational velocity, and where all vectors in the right member are defined at time t. Then Mignard expands V at first order in Δt and derives the force $\mathbf{F}$ and torque Γ undergone by the interacting body:

$$\mathbf{F} = -m'\mathbf{grad}_{\mathbf{r}'} V = 3\frac{Gm^* m' R^5 k_2 \Delta t}{r'^5 r^5} \times$$
$$\Bigg\{ \frac{5}{r'^2}\Bigg[(\mathbf{r}'\cdot\mathbf{r})\big[\mathbf{r}\cdot(\overrightarrow{\omega}\times\mathbf{r}') + \mathbf{r}'\cdot\mathbf{v}\big]$$
$$- \frac{1}{2r^2}(\mathbf{r}\cdot\mathbf{v})\big[5(\mathbf{r}'\cdot\mathbf{r})^2 - r'^2 r^2\big]\Bigg]\mathbf{r}'$$
$$-\big[\mathbf{r}\cdot(\overrightarrow{\omega}\times\mathbf{r}') + \mathbf{r}'\cdot\mathbf{v}\big]\mathbf{r}$$
$$-(\mathbf{r}'\cdot\mathbf{r})\big[\mathbf{r}\times\overrightarrow{\omega} + \mathbf{v}\big] + \frac{\mathbf{r}\cdot\mathbf{v}}{r^2}\big[5(\mathbf{r}'\cdot\mathbf{r})\mathbf{r} - r^2\mathbf{r}'\big]\Bigg\},$$

$$\Gamma = \mathbf{r}' \times \mathbf{F} = -3\frac{Gm^* m' R^5 k_2 \Delta t}{r'^5 r^5} \times$$

$$\left\{ \left[\left[\mathbf{r} \cdot (\overrightarrow{\omega} \times \mathbf{r}') + \mathbf{r}' \cdot \mathbf{v} \right] - 5\frac{\mathbf{r} \cdot \mathbf{v}}{r^2}(\mathbf{r}' \cdot \mathbf{r}) \right] (\mathbf{r}' \times \mathbf{r}) \right.$$

$$\left. +(\mathbf{r}' \cdot \mathbf{r})\left[(\mathbf{r}' \cdot \overrightarrow{\omega})\mathbf{r} - (\mathbf{r}' \cdot \mathbf{r})\overrightarrow{\omega} + \mathbf{r}' \times \mathbf{v} \right] \right\}.$$

The determination of Γ gives the contribution of the body tides to the variation of the spin:

$$\begin{cases} \dfrac{dL}{dt} = -\dfrac{1}{\omega}\Gamma \cdot \overrightarrow{\omega} \\ \dfrac{dX}{dt} = -\Gamma \cdot \mathbf{K} \end{cases}$$

where $\mathbf{K}$ is the normal to the ecliptic.

We have computed Γ with the help of the algebraic manipulator TRIP (Laskar, 1994b), writing all vectors in ecliptic coordinates and averaging both formula over the periods of mean anomaly, longitude of node and perigee of the perturbing body (and of the interacting body if it is not the same one). Taking second order truncations in eccentricities, we obtain:

- the contributions of the solar tides ($\mathbf{r}' = \mathbf{r} = \mathbf{r}_\odot$):

$$\begin{cases} \dfrac{dL}{dt} = -\dfrac{3Gm_\odot^2 R^5 k_2 \Delta t}{2a_\odot{}^6} \times \\ \qquad \left[\left(1 + \frac{15}{2}e_\odot{}^2\right)\left(1 + \dfrac{X^2}{L^2}\right)\dfrac{L}{C} - 2(1 + \frac{27}{2}e_\odot{}^2)\dfrac{X}{L}\, n_\odot \right] \\ \dfrac{dX}{dt} = -\dfrac{3Gm_\odot^2 R^5 k_2 \Delta t}{2a_\odot{}^6} \times \\ \qquad \left[2\left(1 + \frac{15}{2}e_\odot{}^2\right)\dfrac{X}{C} - 2(1 + \frac{27}{2}e_\odot{}^2)n_\odot \right] \end{cases}$$

where $n_\odot$ is the mean motion of the Sun around the Earth.

- the contributions of the lunar tides ($\mathbf{r}' = \mathbf{r} = \mathbf{r}_M$):

$$\begin{cases} \dfrac{dL}{dt} = -\dfrac{3Gm_M^2 R^5 k_2 \Delta t}{2a_M^6} \times \\ \qquad \left[\frac{1}{2}\left(1 + \frac{15}{2}e_M{}^2\right)\left[3 - \cos^2 i_M + (3\cos^2 i_M - 1)\dfrac{X^2}{L^2}\right]\dfrac{L}{C} \right. \\ \qquad \left. -2(1 + \frac{27}{2}e_M{}^2)\dfrac{X}{L} n_M \cos i_M \right] \\ \dfrac{dX}{dt} = -\dfrac{3Gm_M^2 R^5 k_2 \Delta t}{2a_M^6} \times \\ \qquad \left[\left(1 + \frac{15}{2}e_M{}^2\right)(1 + \cos^2 i_M)\dfrac{X}{C} \right. \\ \qquad \left. -2(1 + \frac{27}{2}e_M{}^2)\, n_M \cos i_M \right] \end{cases}$$

- the contributions of the "cross tides" ($\mathbf{r}' = \mathbf{r}_M$ and $\mathbf{r} = \mathbf{r}_\odot$; $\mathbf{r}' = \mathbf{r}_\odot$ and $\mathbf{r} = \mathbf{r}_M$), the cases where the Moon and the Sun are respectively the perturbing bodies accounting for the same quantity:

$$\begin{cases} \dfrac{dL}{dt} = -\dfrac{3Gm_\odot m_M R^5 k_2 \Delta t}{4a_\odot{}^3 a_M{}^3} \times \\ \qquad \left(1 + \frac{3}{2}e_\odot{}^2\right)\left(1 + \frac{3}{2}e_M{}^2\right)(3\cos^2 i_M - 1)\left(1 - \dfrac{X^2}{L^2}\right)\dfrac{L}{C} \\ \dfrac{dX}{dt} = 0 \end{cases}$$

Let us have a look on the consequences of all these contributions. Neglecting the eccentricities, the variation of the obliquity due to the direct solar tides or the lunar ones with a low inclination of the Moon has the form:

$$\frac{dx}{dt} = k(x^2 - 1)(\tfrac{1}{2}\omega x - n)$$

where $x = \cos\epsilon$ and k is a positive quantity. This implies that $\epsilon = 0°$ and $\epsilon = 180°$ are two instable positions of equilibrium and that $\epsilon = \arccos(\frac{2n}{\omega}) = \epsilon_0$ is a stable position, but it is a relative stability because the braking of L makes it slowly moves down to 0°. Furthermore, the obliquity increases when $\epsilon < \epsilon_0$ and decreases otherwise.

It is easy to see that the cross tides drive the equator towards the orbital plane. They are missing in Mignard's articles but Touma and Wisdom (1994) have pointed out their relative importance. Actually, the ratio of their magnitude with the one of the direct solar tides (21.6% of the lunar one) is

$$\frac{m_M a_\odot{}^3}{2m_\odot a_M{}^3} \simeq 1.1\,.$$

As dL/dt is proportional to $\sin^2\epsilon$, this contribution must be taken into account whenever the obliquity reaches high values.

We can derive now from $\mathbf{F}$ the variations of the orbit of the interacting body induced by the tides. They can be obtained by determining the components R', S' and W' of $\mathbf{F}$ in an osculating reference frame. These components write:

$$R' = \frac{1}{\mu r'}\mathbf{F} \cdot \mathbf{r}'\,,$$

$$S' = \frac{1}{H'\mu r'}\mathbf{F} \cdot (\mathbf{H}' \times \mathbf{r}')\,,$$

$$W' = \frac{1}{H'\mu}\mathbf{F} \cdot \mathbf{H}',$$

where μ is the so–called reduced mass of the system Earth-interacting body and $\mathbf{H}'$ the orbital angular momentum of the interacting body:

$$\mu = \frac{m_\oplus m'}{m_\oplus + m'} \quad ; \quad H' = m'n'a'^2\sqrt{1 - e'^2}.$$

The orbital variations are given by the Lagrange equations (see Brouwer and Clemence, 1961):

$$\begin{cases} \dfrac{da'}{dt} = \dfrac{2}{n'\sqrt{1-e'^2}}\Big[R'e'\sin v' + S'\dfrac{a'}{r'}(1-e'^2)\Big] \\ \dfrac{de'}{dt} = \dfrac{\sqrt{1-e'^2}}{n'a'e'}\Big[R'e'\sin v' + S'\big(\dfrac{a'}{r'}(1-e'^2) - \dfrac{r'}{a'}\big)\Big] \\ \dfrac{di'}{dt} = \dfrac{r'\cos w'}{n'a'^2\sqrt{1-e'^2}}\,W' \end{cases}$$

where v', w', i' are respectively the true anomaly, the longitude of perigee and the inclination of the interacting body on the ecliptic E_{c_t}. After expanding and averaging these equations and taking second order truncations in eccentricity, we find for the Moon:

$$\begin{cases} \dfrac{da_M}{dt} = \dfrac{6Gm_M{}^2R^5k_2\Delta t}{\mu a_M{}^7}\times \\ \qquad \Big[\big(1+\tfrac{27}{2}e_M{}^2\big)\dfrac{X}{Cn_M}\cos i_M - (1+23e_M{}^2)\Big] \\ \dfrac{de_M}{dt} = \dfrac{3Gm_M{}^2R^5k_2\Delta t\, e_M}{\mu a_M{}^8}\Big[\tfrac{11}{2}\dfrac{X}{Cn_M}\cos i_M - 9\Big] \\ \dfrac{d\cos i_M}{dt} = \dfrac{3Gm_M{}^2R^5k_2\Delta t}{2\mu a_M{}^8}(1+8e_M{}^2)\dfrac{X}{Cn_M}\sin^2 i_M \end{cases} \tag{5}$$

and for the Sun:

$$\begin{cases} \dfrac{da_\odot}{dt} = \dfrac{6Gm_\odot{}^2R^5k_2\Delta t}{m\_o a_\odot{}^7}\Big[\big(1+\tfrac{27}{2}e_\odot{}^2\big)\dfrac{X}{Cn_\odot} - (1+23e_\odot{}^2)\Big] \\ \dfrac{de_\odot}{dt} = \dfrac{3Gm_\odot{}^2R^5k_2\Delta t\, e_\odot}{m_\odot a_\odot{}^8}\Big[\tfrac{11}{2}\dfrac{X}{Cn_\odot} - 9\Big] \end{cases}$$

but both last variations are negligible: about 3 meters per Myr for $da_\odot/dt$ and 10^{-12} per Myr for $de_\odot/dt$.

Two contributions are missing so far: the tides raised on the Moon by the Earth and by the Sun. With the following assumptions:

a) The inclination of the Moon's equator on its orbital plane is small (6.41° at this time), so we can use the approximations $\epsilon_M \simeq 0$ and $i' \simeq 0$,

b) The Moon is locked in synchronous spin–orbit resonance 1 : 1 with the Earth (i.e. $\omega_M = n_M$),

the tide raised by the Moon on the Earth is obtained by exchanging Moon and Earth in Eqs. (5) with the above simplifications. We obtain the additional contributions:

$$\begin{cases} \dfrac{da_M}{dt} = -\dfrac{57Gm_\oplus{}^2R_M{}^5k_{2M}\Delta t_M e_M{}^2}{\mu a_M{}^7} \\ \dfrac{de_M}{dt} = -\dfrac{21Gm_\oplus{}^2R_M{}^5k_{2M}\Delta t_M e_M}{2\mu a_M{}^8} \end{cases}$$

If one takes $k_{2M}\Delta t_M \simeq 213$ suggested by the DE245 data, $k_2 = 0.305$ (Lambeck, 1980), and $\Delta t = 638$ seconds, these contributions represent 1.2% of the total da_M/dt and 30% of the total de_M/dt for present conditions. As Mignard pointed it out, the terrestrial tides on the Moon have no substantial effect on the lunar orbit unless it is close to the Earth (at a few Earth's radii) and when the ratio $k_{2M}\Delta t_M/(k_2\Delta t)$ is much greater than 1 (using DE245 data, we have $\rho \simeq 1.1$).

Finally, the tides raised on the Moon by the Sun can also be neglected because the ratio of the magnitudes of solar and terrestrial tides on the Moon is

$$\Big(\frac{m_\odot}{m_\oplus}\Big)^2\Big(\frac{a_\odot}{a_M}\Big)^6 \simeq 3.2\times 10^{-5}.$$

3.2. The core–mantle friction

Here we basically rely on Rochester's model (1976).

The inner Earth is composed of a mantle and a core separated into a central rigid part and a fluid one. We neglect interactions between both last parts because they are supposed to be strongly coupled by pressure forces (Hinderer, 1987). The core and the mantle have different dynamical ellipticities, so they tend to have different precession rates. This trend produces a viscous friction at the core–mantle boundary (CMB). Thus, there are motions in the outer liquid core inducing electric currents which generate a torque of electromagnetic friction because of magnetization of the deepest layer of the mantle.

Rochester showed that the magnetic friction has only a faint effect on the Earth's long term rotational dynamics. If the coefficient of viscous friction due to the viscosity of the liquid metal of the outer core is thought weaker that the magnetic one, some turbulences and inhomogeneïties in the outer core could make the viscous friction far more efficient (Lumb and Aldridge, 1991), (Williams, 1993), so that we will suppose that the friction is solely viscous. We will consider here that an effective viscosity ν can account for a weak laminar friction, as well as a strong turbulent one which thickens the boundary layer.

Two additional torques account for the coupling: the inertial torque **N** due to pressure forces at the CMB (which is not spherical due to the Earth's rotation) , and the topographic torque (Hide, 1969) due to likely "bumps" of this boundary. **N** tends to attach strongly the core and the mantle but its effect is reduced if the boundary layer beneath the CMB is thick. Although irregularities of the CMB increase the surface of friction, this topographic torque (which is not taken into account in Rochester's model) would rather have a conservative effect, the bumps acting as notches; in this way it can be considered as the irregular part of **N**. Besides, it is still too poorly known to estimate its long term contributions (Jault, *private communication*). We will also ignore it.

The angular momentum theorem applied to the core (subscript c) and to the mantle (subscript m) then gives:

$$\begin{cases} \dfrac{d(C_c\overrightarrow{\omega_c})}{dt} = \mathbf{P}_c + \mathbf{N} + \mathbf{F} \\ \dfrac{d(C_m\overrightarrow{\omega_m})}{dt} = \mathbf{P}_m - \mathbf{N} - \mathbf{F} \end{cases} \tag{S}$$

where $\mathbf{P}_c$, $\mathbf{P}_m$ are the precession torques, and $\mathbf{F}$ the frictional torque. C_c and C_m denote the moment of inertia with $C = C_c + C_m$. In first approximation, one can write:

$$\mathbf{F} = \kappa(\overrightarrow{\omega_m} - \overrightarrow{\omega_c}) = \kappa\,\overrightarrow{\delta}$$

where κ is the effective coefficient of friction. Rochester gives the following approximation to the solution of system (S):

$$\frac{d\epsilon}{dt} = -\frac{\kappa\dot{\psi}^2 \sin\epsilon\cos\epsilon}{\gamma_{el} {E_{d_c}}^2 C\omega^2}$$

where γ_{el} is the elasticity correcting factor of the mantle and $\dot{\psi}$ the Earth's rate of precession. E_{d_c} is the dynamical ellipticity of the core which, as the whole body one, is assumed to be proportional to the square of the speed of rotation. It should not differ from the value corresponding to the hydrostatic equilibrium by more than a few percents (Hinderer, 1987). Hence $E_d = E_{d_{in}}\omega^2/{\omega_{in}}^2$ and $E_{d_c} = E_{d_{c_{in}}}\omega^2/{\omega_{in}}^2$. where the subscript *in* denotes the initial value. As pointed out by Yoder (1995), such approximations would not be valid any more for a slow rotation, for which the non-hydrostatic parts of the ellipticities can dominate.

This solution is valid only when the inertial torque $\mathbf{N}$ is non–zero. This happens when the ellipticity of the CMB exceeds the ratio of the rotation period to the precession period (Poincaré, 1910, see also Peale, 1976). Assuming that this ellipticity is roughly in hydrostatic equilibrium, this condition is equivalent to:

$$\omega > \left(\pi G\rho_c \mid \dot{\psi} \mid\right)^{1/3}, \qquad (C)$$

where ρ_c is the density of the outer core. The rate $\dot{\psi}$ being proportional to ω (see Sect. 2), such a condition is satisfied since ω exceeds a given constant; this is the case for a fast–rotating planet like the Earth.

We determine κ as a function of the effective viscosity with the help of Goldreich and Peale (1967, 1970). First, we compute the "spin-up" time which corresponds to the time that the core needs to adjust its rotation to the one of the mantle in absence of any external force. This is the characteristic time τ of the following system:

$$\begin{cases} \dfrac{d(C_m\overrightarrow{\omega_m})}{dt} = -\mathbf{F} = -\kappa\,\overrightarrow{\delta} \\[2ex] \dfrac{d(C_c\overrightarrow{\omega_c})}{dt} = \mathbf{F} = \kappa\,\overrightarrow{\delta} \end{cases}$$

The solution is

$$\overrightarrow{\delta} = \overrightarrow{\delta_0}\, e^{-t/\tau}\ ,\ \tau = \frac{C_c C_m}{\kappa C}.$$

Thus, according to Greenspan and Howard (1963),

$$\tau = \frac{R_c}{\sqrt{\nu\omega}}.$$

Hence

$$\kappa = \frac{C_c C_m\sqrt{\nu\omega}}{CR_c}$$

what yields

$$\frac{d\epsilon}{dt} = -\frac{C_c C_m\dot{\psi}^2\sqrt{\nu}\cos\epsilon\,\sin\epsilon}{\gamma_{el}C^2R_c {E_{d_c}}^2\omega^{3/2}}$$

i.e.

$$\frac{d\cos\epsilon}{dt} = \frac{C_c C_m\dot{\psi}^2\sqrt{\nu}}{\gamma_{el}\sqrt{C}R_c {E_{d_c}}^2}\left(1 - \frac{X^2}{L^2}\right)\frac{X}{L^{5/2}}.$$

The contributions of the core–mantle friction to dL/dt and dX/dt are obtained thanks to the fact that $\overrightarrow{\omega_c}$ is very close to $\overrightarrow{\omega_m}$ because of the action of $\mathbf{N}$. Then the precession torques $\mathbf{L}_m$ and $\mathbf{L}_c$ belonging to the orbital plane of normal $\mathbf{K}$ (this clearly appears when the torques are expressed in a vectorial form; see for example Goldreich 1966), the addition of both equations of (S) and the scalar product by $\mathbf{K}$ of the resulting equation leads to

$$\frac{dX}{dt} \simeq 0,$$

which implies

$$\frac{dL}{dt} = -\frac{C_c C_m\dot{\psi}^2\sqrt{\nu}}{\gamma_{el}\sqrt{C}R_c {E_{d_c}}^2}\left(1 - \frac{X^2}{L^2}\right)\frac{1}{L^{1/2}}.$$

This quantity being always negative, the core–mantle friction (CMF) tends to slow down the rotation and to bring the obliquity down to 0° if $\epsilon < 90°$ and up to 180° otherwise, what contrasts with the effect of the tides. Furthermore, one can see that, despite the strong coupling by pressure forces, there can be a substantial contribution to the variation of the spin for high viscosities and moderate speeds of rotation. It must be pointed out that this contribution has no sense for an arbitrary high viscosity (which would attach the core to the mantle) and for an arbitrary small core dynamical ellipticity (for which $\mathbf{N}$ vanishes).

Finally, the equation $dX/dt = 0$ has also the following remarkable consequence: since

$$\frac{d\cos\epsilon}{dt} = \frac{1}{L}\left(\frac{dX}{dt} - \frac{X}{L}\frac{dL}{dt}\right),$$

we have

$$\frac{dL}{L} = -\frac{d\cos\epsilon}{\cos\epsilon}$$

what yields

$$\int_{t_1}^{t_2}\frac{d\omega}{\omega} = -\int_{t_1}^{t_2}\frac{d\cos\epsilon}{\cos\epsilon}$$

i.e.

$$\frac{\omega(t_1)}{\omega(t_2)} = \frac{\cos\epsilon\,(t_2)}{\cos\epsilon\,(t_1)} \qquad (R)$$

This simple relation is independent of the variations of ν and strongly constraints the possible evolution of ϵ.

3.3. The atmospheric tides

The Earth's atmosphere also undergoes some torques which can be transmitted to the surface by friction: a torque caused by the gravitational tides raised by the Moon and the Sun, a magnetic one generated by interactions between the magnetosphere and the solar wind. Both effects are negligible; see respectively Chapman and Lindzen (1970) and Volland (1988).

Finally, a torque is produced by the daily solar heating which induces a redistribution of the air pressure, mainly driven by a semidiurnal wave, hence the so-called thermal atmospheric tides (Chapman and Lindzen, 1970). The axis of symmetry of the resulting bulge of mass is permanently shifted out of the direction of the Sun by the Earth's rotation. As for the body tides, this loss of symmetry is responsible for the torque which, at present conditions, tends to accelerate the spin.

Volland (1988) showed that this effect is not negligible, reducing the Earth's despinning by about 7.5%. But, although the estimate of their long term contributions surely deserves careful attention, we have not taken the atmospheric tides into account in the computations presented in the next sections, assuming that, as the uncertainty on some of the other factors is still important, the global results obtained here will not differ much when taking this additional effect into consideration.

4. Some limits to the coefficients of dissipation

Goldreich (1966), Mignard (1979, 1980, 1981) and Touma and Wisdom (1994) have studied the past evolution of the Earth–Moon system taking the distance of the Moon to the Earth as the independent variable instead of the time. This is because if one takes the present value $\Delta t = 638$ seconds which fits the observed receding of the Moon for which Dickey et al. (1994) give 3.82 cm per year, it is found that the capture or formation of the Moon occurred at about -1.2 Gyr which contradicts most of paleogeological observations (see for example Piper, 1978, Lambeck, 1980).

So Δt has clearly been smaller in the past, probably because of the changes in the continental distribution (Krohn and Sündermann, 1978) and in the oceanic loading during glaciations. We only need for our purpose to consider some acceptable average value anyway.

With a review of experiments in laboratories and observations of fluctuations of the Earth's spin, Lumb and Aldridge (1991) give a range of possible values for the effective viscosity ν going from 10^{-7} up to $4.6 \times 10^5\,\mathrm{m^2s^{-1}}$. Although the influence of the friction is proportional to $\sqrt{\nu}$, the uncertainty about the effect of internal friction still remains a serious problem. Rochester has chosen to set the upper limit to $10\,\mathrm{m^2s^{-1}}$ which comes from observations of the fluctuations of the Earth's nutation (Toomre, 1974). Actually this last value changes the spin very little. On the opposite, Williams asserts that ν should be much larger. This comes out from his assessment of observations of sediments and fossils which suggests that the history of the obliquity would have been very different from the consensual one, starting from 70° and going down to the present 23°27′ with a drastic falldown at about -630 Myr (Williams, 1993).

A simple way to determine some strong constraints, provided that Rochester's model is assumed always valid, is to look for the couples $(\Delta t, \nu)$ which give, with the present model, an evolution of the length of the day (LOD) similar to the ones given by the observations of the Earth's ground over the last two billion years (Williams, 1989).

The data gathered by Williams (1989) are of distinct origins and they give some quite different rates of increasing of the LOD, especially for the first hundreds Myr. We have decided to take as references two of these rates. The first one is based on the maximum values of the LOD at about -500 Myr which corresponds to a 20.33 hour rotation period. This gives the ratio $\beta(-500\,\mathrm{Myr}) = \omega(-500\,\mathrm{Myr})/\omega(0) \simeq 1.18$. Such a choice is supported by the sake of getting upper bounds for the dissipation coefficients. The second rate is based on the -2 Gyr data and the observation of Elatina formation at -650 Myr. It corresponds to a 19 hour rotation period at -2 Gyr which gives $\beta(-2\,\mathrm{Gyr}) \simeq 1.26$. This one is suggested by the fact that most ancient observations match better with the Moon's orbital history and corresponds to a low rate of braking.

We have computed the speed of the Earth's rotation at -500 Myr and -2 Gyr for 22 values of Δt, going from 0 to 630 seconds by steps of 30 seconds, and for 25 values of ν, going from 0 to $24^3 = 13824\,\mathrm{m^2s^{-1}}$ by cubic steps.

The following physical parameters of the present Earth appearing in the previous equations have been taken from Lambeck (1988):

$$k_2 = 0.305,$$
$$C_m = 0.9\,C,$$
$$C_c = 0.1\,C,$$
$$R_c = 3.47\ 10^{-6} m,$$
$$E_{dc} = 2.5\ 10^{-3},$$

and $\gamma_{el} = 0.57$ (Legros, *private communication*). All other parameters and initial conditions are taken from Laskar (1986).

The results are shown in Figs. 3a-b where $\beta(t)$ is given for a wide range of $(\Delta t, \nu)$. In the computation at -2 Gyr, Δt only goes up to 360 seconds because higher values have led to the collision with the Moon before this date. In Fig. 3 we have superimposed the curves corresponding to the ratios $\beta(-500\,\mathrm{Myr})$ and $\beta(-2\,\mathrm{Gyr})$. For the case -500 Myr, we have also drawn the curve of $\beta(-500\,\mathrm{Myr}) = 1.04$ which corresponds to the lower observed rate of deceleration, so that the strip formed by both curves represents the global uncertainty on the couple $(\Delta t, \nu)$.

It is clear that 600 seconds is an upper limit for average Δt. The observations at -2 Gyr set this limit at about 200 seconds. Moreover, the upper limit for average ν (with no tidal effects) is about $7400\,\mathrm{m^2s^{-1}}$, and we see that the core–mantle friction has no significant effect below several $\mathrm{m^2\,s^{-1}}$. The observations at -2 Gyr lead to $\nu_{max} \simeq 1000\,\mathrm{m^2s^{-1}}$.

It would be interesting to know the lower limit of Δt, which should be the value of the mantle alone because most of the fluctuations comes from the changes in oceanic loading. It is

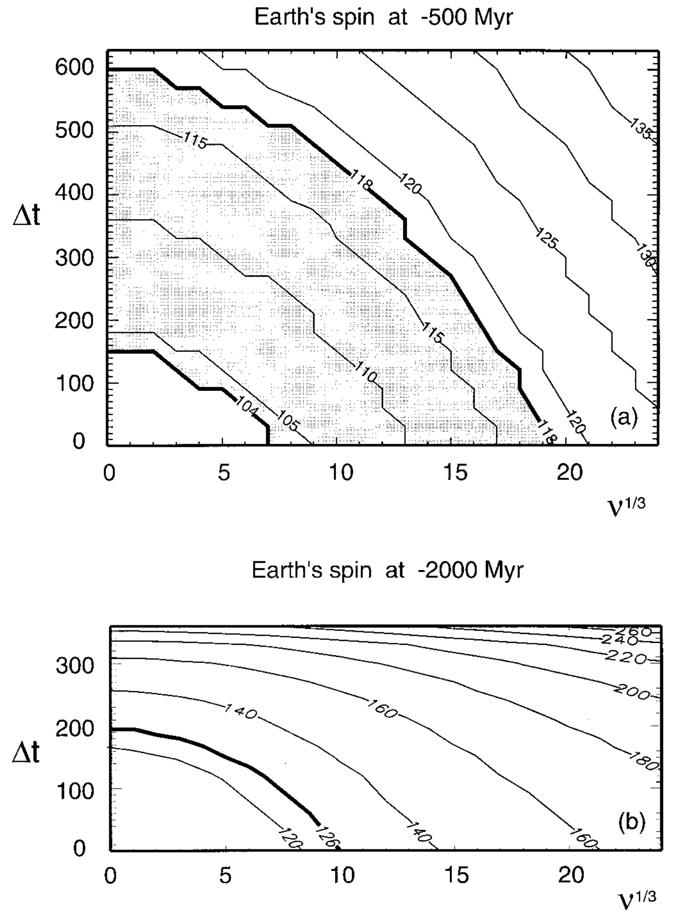

Fig. 3. a Percentage of the ratio of the speed of rotation at −500 Myr over the present one for various values of the tidal delay Δt and the viscosity ν. The two bold lines delimit an acceptable range, in agreement with the observations from sediments and fossils. **b** Same percentage at −2 000 Myr. The bold line corresponds to the observation of Williams (1989).

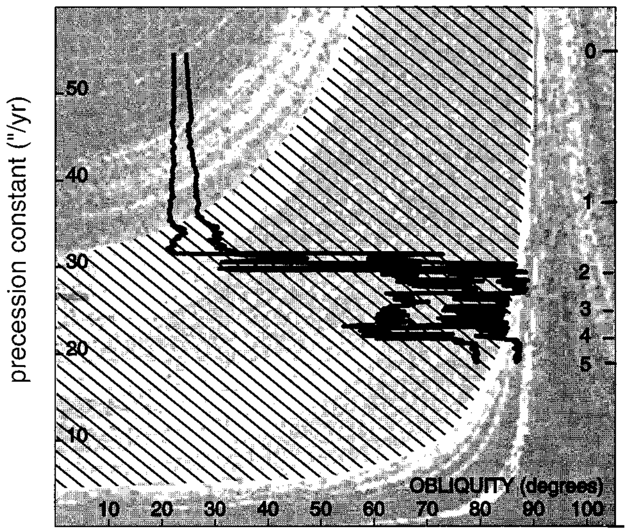

Fig. 4a. Example of possible evolution of the Earth's obliquity for 5 Gyr in the future, for $\Delta t = 600$ s. The background of the figure is the same one as in Fig. 1, and is a global view of the stability of the obliquity, obtained by means of frequency map analysis (see Laskar and Robutel, 1993). The precession constant (on the left) is plotted against the obliquity: the two bold curves correspond to the minimum and maximum values reached by the obliquity. The right y-axis gives the corresponding time for the motion. The non-hatched zone corresponds to very regular regions, and we actually observe that in these regions, the motion suffers only small (and regular) variations. The hatched parts are the regions of strong chaotic behavior. Indeed, in the present simulation, as soon as the orbit enters this chaotic zone, very strong variations of the obliquity are observed, and very high values, close to 90 degrees, are reached.

possible to give an rough estimate to it, knowing that the energy dissipated in the oceans accounts for about 90 or 95% of the total (Zschau, 1978), (Cazenave, 1983), (Mignard, 1983), (Lambeck, 1988). In this case, the lowest Δt would equal 30 or 60 seconds, hence a largest ν of about 600 or 800 m^2s^{-1} if one relies on the −2 Gyr observations, and about 4400 or 4700 m^2s^{-1} for the −500 Myr ones.

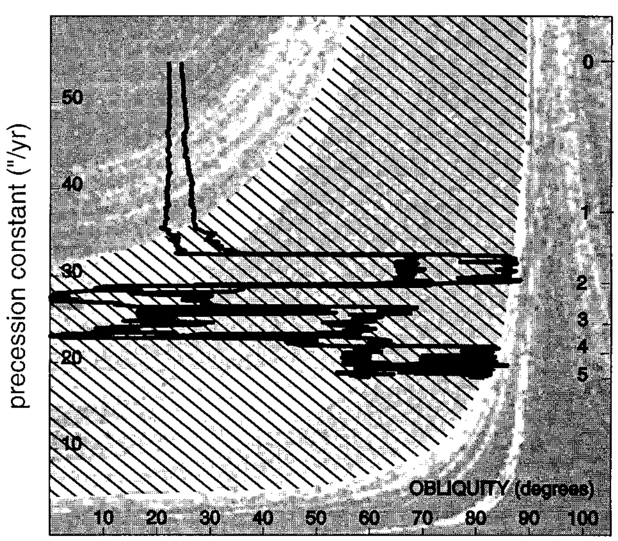

Fig. 4b. Same as Fig. 4a, but with a difference of 10^{-8} degree in the initial obliquity.

5. Williams' scenario for the history of the Earth's obliquity

The dissipation mechanisms presented therein give us some constraints on scenarios of the Earth's evolution, and our aim here would be to provide a general framework in which all scenario for the evolutions of the Earth's obliquity should be described. As an example, we show here that the dynamical constraints obtained here allow to question the scenario proposed by Williams (1993). Interpreting observations of various deposits in the Earth's soil which depend on weathering conditions, Williams devised the following scenario for the past evolution of the Earth's obliquity:

a) a slow and regular decreasing from 70° to 60° between −4.5 Gyr and −630 Myr;

b) a quick falldown from 60° to 26° between −630 and −220 Myr;

c) a slow decreasing till the present value.

Concerning the first stage, the main objection to such a smooth evolution arises from the fact that a 70° obliquity would imply a crossing of the chaotic zone (see Fig. 1), hence strong fluctuations ranging from about 65 degrees to about 90 degrees (Laskar *et al.*, 1993b).

Then, we have estimated the value of ν which would correspond to the second and third ones. The slow decreasing of 2.5° during the last 430 Myr might be possible, the corresponding ν being about 300 m^2s^{-1}. Now, a falldown from 60° to 26° within 220 Myr gives, with $\Delta t = 200$ seconds, a huge value of $1.3 \times 10^6\, m^2s^{-1}$ which exceeds the upper limit of Lumb and Aldridge (1991). Such a viscosity would strongly slow down the Earth and the corresponding LOD in the past would be very far from the observed one with $\beta(-220\text{ Myr}) \simeq 1.72$.

More simply and independently of the problem of the possible evolution of the value of ν, it is straightforward to verify that the variations proposed by Williams do not respect relation (*R*). Indeed, as

$$\frac{\cos(26°)}{\cos(60°)} \simeq 1.8,$$

we should have $\omega(-630\text{Myr}) = \omega(-220\text{Myr}) \times 1.8$ which does not correspond to any plausible despinning factor even during the whole last Gyr.

Williams found a support to his assessment in the very large rate of $d\epsilon/dt = -0.1''\text{cy}^{-1}$ (Kakuta and Aoki, 1972) due to core–mantle coupling. One one hand, as Rochester (1976) noticed it, this value was based on a model which is irrelevant since it does not take into account the inertial coupling; Aoki's model (Aoki, 1969) is adapted only to a slow–rotating planet like Venus at present time for which $d\omega/dt$ is proportional to $\nu^{-1/2}$ (if ν is not too small). On the other hand, Aoki's model also verifies relation (*R*) — which does not depend on Rochester's approximations —, and such a rate for $d\epsilon/dt$ does not correspond to the low rate of braking $\dot{\omega}/\omega = -5.8 \times 10^{-14}$ proposed by Aoki and Kakuta (1971); Williams thought this last rate was coherent with the loss of rotational kinetic energy due to CMF estimated by Rochester, but he did not take the right term of this loss to compare with.

Williams mentions that some “special conditions” should have occurred at the CMB in order to explain the drastic falldown. He also suggests that a resonance between the free core nutation and the retrograde annual nutation caused by the solar torque may have played an important role. Climate friction (Bills, 1995) might also be a candidate for additional variations of the obliquity. Such effects remain uncertain. Characteristics of the Earth's interior may have been somewhat different in a remote past, but unless system (*S*) has been very incomplete for some time in the last Gyr, his scenario should be rejected.

6. the next five billion years evolution of the Earth–Moon system

As we have managed to set up some limitations on the possible values of the tidal dissipation and the viscosity of the outer core, by using the available geological observations of the past evolution of the Earth, we are now ready to study its future over its expected lifetime, i.e. about 5 Gyr.

6.1. with the present tidal coefficient and no core–mantle friction

Using Laskar's theory of the solar system, we simultaneously integrate over 5 Gyr the motion of all the planets (Pluto is not taken into account) and the angular momenta of the Earth and the Moon with a 250 yr time step.

The equations for the planetary orbital motion used here are the averaged equations which were previously used by Laskar for the demonstration of the chaotic behavior of the solar system. They include the Newtonian interactions of the 8 major planets of the solar system (Pluto is neglected), and relativistic and Lunar corrections (Laskar, 1985, 1989, 1990). The numerical solution of these averaged equations showed excellent agreement when compared over 4400 years with the numerical ephemeris DE102 (Newhall *et al.*, 1983, Laskar, 1986), and over 3 millions years with the numerical integration performed by Quinn, Tremaine and Duncan (Quinn *et al.*, 1991, Laskar *et al.*, 1992). Similar agreement was observed with subsequent numerical integration by Sussman and Wisdom (1992).

This system of equations was obtained with dedicated computer algebra and contains about 50000 monomial terms of the form $\alpha z_1 z_2 z_3 z_4 z_5$ (Laskar, 1985). It was first constructed in a very extensive way, containing all terms up to second order with respect to the masses, and up to 5th degree in eccentricity and inclination, which led to 153824 terms, and then truncated to improve the efficiency of the integration, without significant loss of precision (Laskar, 1994). The numerical evaluation of this simplified system is very efficient, since fewer than 6000 monomials need to be evaluated because of symmetries. Numerical integration is carried out using an Adams method (PECE) of order 12 and with a 250-year stepsize. The integration error was measured by integrating the equations back and forth over 10 Myr. It amounts to 3×10^{-13} after 10^7 years (40 000 steps), and behaves like $t^{1.4}$. Ignoring the chaotic behavior of the orbits, this would give a numerical error of only 4×10^{-9} after 10 Gyr.

It is clear that because of the chaotic dynamics with a characteristic time of 5 Myr, the orbital solution loses its accuracy beyond 100 Myr. This is not important since we do not look for the exact solution, but for what happens when the system enters the chaotic zone; the fact that this zone may not be at the exact location has not much importance.

We have chosen to take the near present value of 600 seconds for Δt and no core–mantle friction. In order to have a statistical view of the possible behaviors, the whole system is simultaneously integrated over 5 Gyr for 500 different initial orientations with obliquities very close to ϵ_{J2000} (23° 26′ 21.448″): 10 ini-

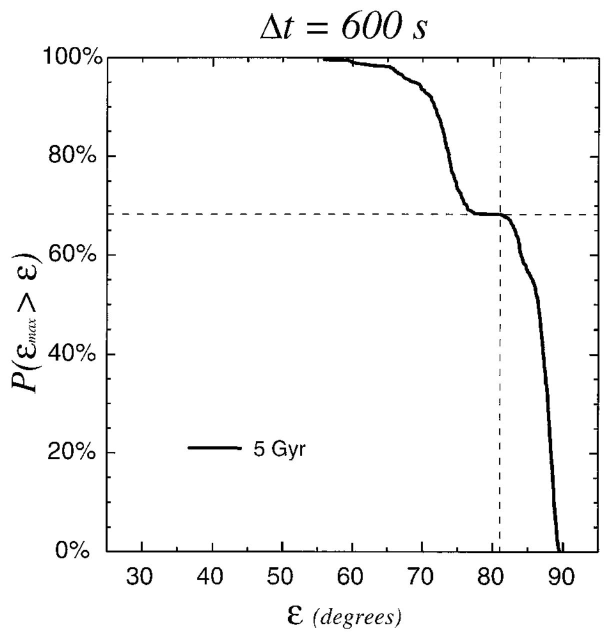

Fig. 5. Probability P for the maximum obliquity ϵ_{max} to exceed a given value ϵ for the Earth with $\Delta t = 600s$. This was performed over 500 orbits with very close initial conditions followed over 5 Gyr.

tial phases ψ separated by 10^{-9} rad and 50 initial obliquities separated by 10^{-8} degree.

Thus we have performed a frequency analysis (see Laskar, 1993) on the precession frequency and also plotted the minimum and maximum reached obliquity each 10.26 Myr. The whole computation, for such an experiment, took about 13 days on a IBM-RS6000/390.

It is quite obvious that we cannot display all the various solutions, and we just selected two examples of the possible evolution of the Earth (Fig.4a-b) which are representative of the whole experiment. The two curves plotted in Figs. 4a and 4b have initial obliquities differing by 36 μas. We see that the obliquity enters the chaotic region at about $+1.5$ Gyr and that it can go from $0°$ to values close to $90°$ as was the case in the conservative framework. When superimposed on Fig. 1, those graphs show possible paths of the evolving obliquity through the different zones of the global dynamics.

The computed speed of rotation of the Earth after 5 Gyr is about 0.42 ω_{in}. Provided that $\rho_c \simeq 10$ kg m^{-3} (Hinderer et al., 1990) and that $\dot{\psi}(5Gyr) < \dot{\psi}_{in}$, one can easily check that condition (C) of Sect. 3 has not been violated.

The 500 different paths obtained in this manner allow us to get a fairly good idea of the probability for the obliquity to attain some given threshold once the chaotic zone entered. For instance, we have found that 342 maximum obliquities have exceeded $81°$ at least once, hence a probability $P(\epsilon > 81°) = 68.4\%$ (see Fig. 5).

6.2. some alternatives

• $\Delta t = 600$ seconds is close to the present measured value of the dissipation coefficient, and is in agreement with the observations at -500 Myr (Fig. 3a), but this leads to a lunar collision at about 1.2 Gyr in the past. For this reason, we also considered for Δt the smaller value of 200 seconds which is close to the lowest value compatible with these geological observations (Fig. 3a). As previously, for $\Delta t = 200$ seconds, we followed the evolution of 500 obliquities. As the dissipation is three times weaker, the Earth reaches the chaotic zone on a much longer time, after about 4.5 Gyr, and after 5 Gyr it has spent only about 500 Myr in this chaotic zone; the probability of reaching a given high value of obliquity is then lower than in the previous case of $\Delta t = 600$ seconds for which the same situation lasted 3.5 Gyr (see Fig. 6), and we have $P(\epsilon > 81°) = 36.6\%$, which nevertheless is not a small value. If we continue the integrations over 6 Gyr, which is still a possible future lifetime for the Earth, we obtain for $P(\epsilon > 81°)$ the much higher value of 60.4% (Fig. 6). We carried on the computation till 8 Gyr in order to look at the evolution of this probability, and we also plotted in Fig. 6 the corresponding curves for 7 and 8 Gyr. Then, the set of the four curves shows that the longer the Earth remains in the chaotic zone, the higher are the probabilities for the maximum obliquity to reach any value (the possible maximum hardly exceeding $90°$ after 8 Gyr).

We can thus conclude that for any value of the tidal dissipation compatible with the geological observations depicted in Fig. 3a, a very large obliquity in the future of the Earth is a highly probable event.

Finally, we notice that all curves present a falldown at about $70°$ and a step till a second falldown to 0 close to $90°$. This can be understood by the fact that, as is shown in Laskar *et al.*(1993b), the chaotic zone is divided into two regions of strong overlap of secular resonances. In each of these regions, the diffusion of the orbits is rapid, but the connection between these two boxes is more difficult. As soon as a given orbit enters the second box, related to high values of the obliquity, it will rapidly describe it entirely, so we observe in this case a jump in the maximum value reached by the obliquity.

• One would like to consider some very larger coefficients Δt or ν in order to accelerate the effect of the dissipation and to shorten a lot the time of integration by the way. For example, Touma and Wisdom (1994) set a tidal effect about 4000 times stronger than the present value in their study of the past evolution of the Earth's obliquity. We have integrated the system with three different values: $\Delta t = 3 \times 10^4, 3 \times 10^5$, and 3×10^6 seconds, the last one roughly corresponding to what Touma and Wisdom took. The equivalent despinning of the Earth is then respectively achieved after 100 Myr, 10 Myr and 1 Myr instead of 5 Gyr.

The results clearly show that the dynamics are altered as much as the time scale of braking is reduced (see Figs. 7a-c). In the first case, we have found $P(\epsilon > 81°) = 1.2\%$. In the second one, the obliquity remains confined below $47.3°$. Finally, secular resonances have a faint effect in the last case, the obliquity never

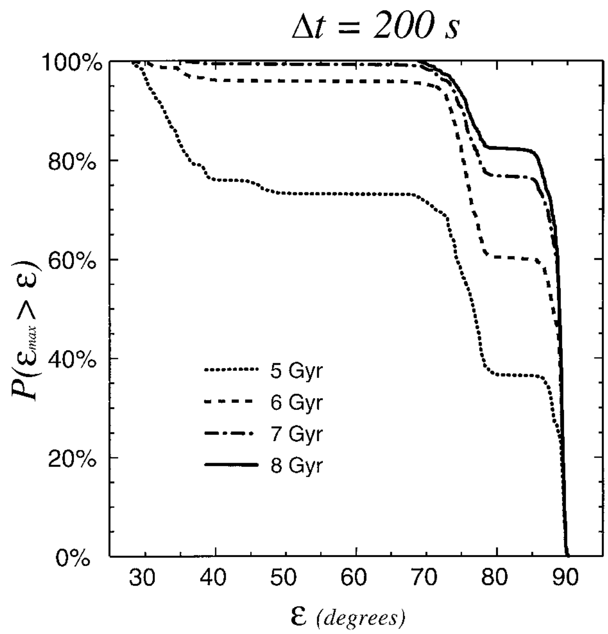

Fig. 6. Probability P for the maximum obliquity ϵ_{max} to exceed a given value ϵ for the Earth with $\Delta t = 200s$ after 5, 6, 7 and 8 Gyr.

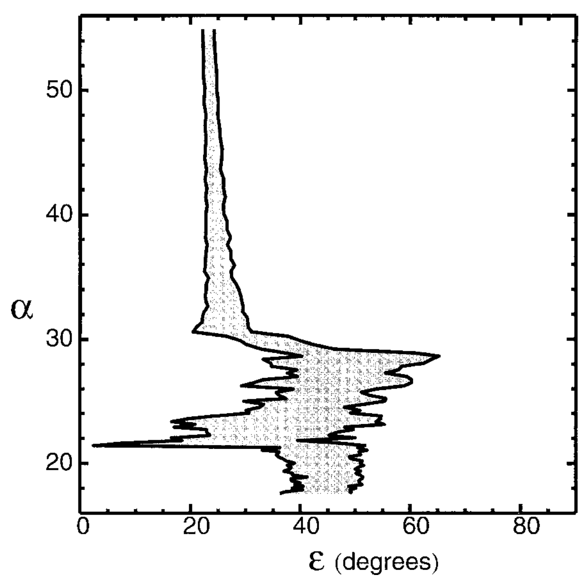

Fig. 7a. as in Fig. 4ab, the precession constant (on the left) is plotted against the obliquity: the two bold curves correspond to the minimum and maximum values reached by the obliquity. Example of evolution for the obliquity of the Earth with $\Delta t = 30000s$.

exceeding 43.5°. For both last cases, the 500 initial conditions nearly give the same evolution.

It is then clear that such a strategy has to be excluded: the time scale of action of the dissipation must be of the same order as the true one.

• We have chosen to neglect the core–mantle friction because the tidal effects are traditionally considered as the only substantial effects. We could also have undertaken some integra-

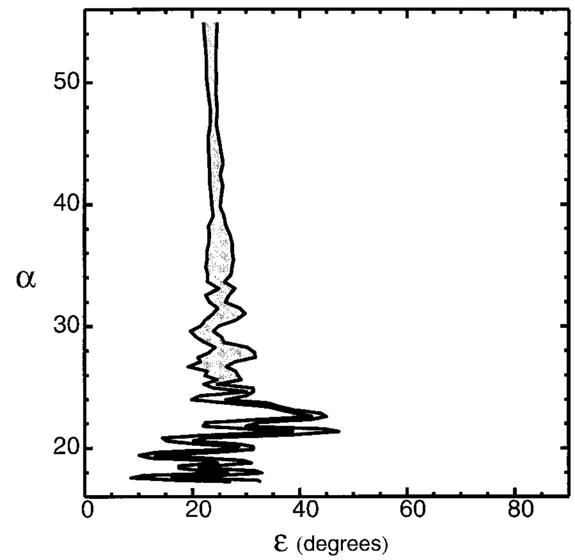

Fig. 7b. Example of evolution for the obliquity of the Earth with $\Delta t = 300000s$.

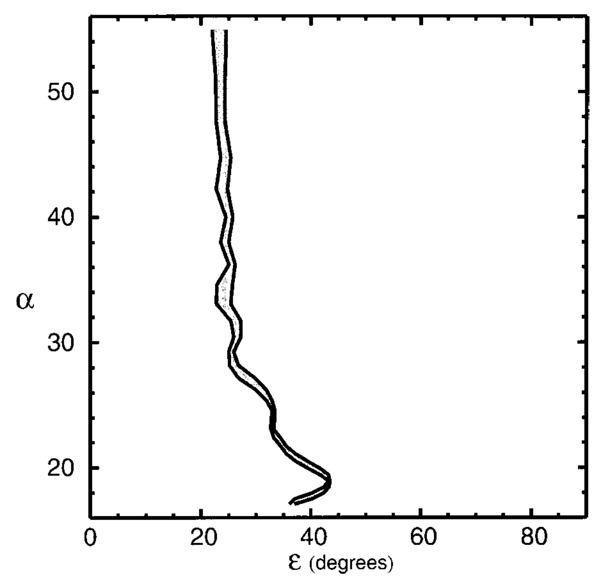

Fig. 7c. Example of evolution for the obliquity of the Earth with $\Delta t = 3000000s$.

tions with non–zero viscosities in order to see the core–mantle friction play an important role, the obliquity being slowly decreasing. As was discussed in Sect. 4, such a scenario is plausible. In any case, the rate of change in obliquity would be relatively small according to relation (R). As the Earth enters the chaotic zone before +4.5 Gyr and undergoes strong variations due to the planetary perturbations, the choice of the couple $(\Delta t, \nu)$ has finally not very much consequence on the very long term evolution of the spin, provided the observations and the time scale of dissipation are respected, according to Fig. 3.

Nevertheless, it must be stressed that for very high viscosities, the CMF could dominate the evolution of the obliquity before the 5 Gyr term. With the highest possible value $\nu = 7400$ m^2s^{-1} suggested by the palaeo-observations, $d\epsilon/dt$ reaches only $-16°$/Gyr when $\omega = 0.5\,\omega_{in}$. Then, such a situation appears to be extreme; this supports our studies with $\nu = 0$.

• The formulation of the tidal dissipation used in these integrations corresponds to a model for which the function Q is inversely proportional to the speed of rotation. Consequently, as suggested by MacDonald's computations (MacDonald, 1964) with a constant geometrical phase lag and the present tidal dissipation rate, the choice of the model for which Q is constant would lead to a faster despinning in the future, and the chaotic zone would be attained sooner.

7. Conclusion

By adding the non-conservative effects to the previous model for the long time evolution of the obliquity of the Earth used by Laskar *et al.*, (1993ab), we possess now a complete model for the study of the long term variations of the spin of the Earth and of the orbit of the Moon over time scales comparable to the age of the Solar System. It has allowed us to find some significant constraints on the poorly known tidal time lag Δt and effective viscosity of the outer core ν thanks to paleo–observations, in spite of the uncertainty in the interpretation of geological data. Any further improvement in the knowledge of one of these two quantities would directly induce an improvement for the second one and in the history of the Earth's spin by the way.

It appears in this study that the action of dissipative effects is weak enough not to cause very significative changes in the behavior of the obliquity compared to one could expect in a conservative framework, so to say that with a rough idea of the time scale of these effects, most of the essential dynamics could be described from Laskar *et al.*(1993b), Laskar and Robutel (1993). The combination of this model with Laskar's secular theory of the solar system provides us with a powerful tool for exploring plausible scenarii for the long term evolution of the obliquity of the terrestrial planets, and reinforces the importance of having a global view of the planetary dynamics.

It is still remarkable that in most cases, when using acceptable rates for the dissipation, the obliquity of the Earth explores a large part of the chaotic region discovered by Laskar *et al.*(1993b), reaching very high maximum values, close to 90 degrees.

The chaotic behavior of the obliquity of the Earth prevented us to describe precisely the evolution of its spin over its age, but here we have shown in a simple probabilistic manner that the most probable destiny of the Earth is to undergo very strong variations of its obliquity before the inflation of the Sun, if it does not occur.

These computations also confirms that in absence of the Moon, that is for precession constants of the order of 20 arcsec/year, the Earth would suffer very large variations of its obliquity, which could reach nearly 90 degrees with a high probability.

Finally, it should be noted that the chaotic regions for the obliquity of Venus (Laskar and Robutel, 1993) is very similar to the one of the Earth (Fig. 1), and thus similar behavior probably occurred for this planet in the past. We are presently undertaking similar computations for this planet, for which a supplementary difficulty consist in a precise understanding of the possible strong effect of the atmospheric tides.

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