

Corrigendum to *Long-term evolution of the Galilean satellites: the capture of Callisto into resonance* by Lari, Saillenfest and Fenucci, A&A 639, A40, 2020

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In the published version of the article, the method used to compute the rotation vector Θ of the non-inertial reference frame (Appendix B.3) is erroneous. Strictly speaking, the method used in the published version is correct only if there is a single rotation between the two reference frames (e.g. about the vertical axis). Since both the obliquity and orbital inclination of Jupiter are small, the method used in the published version does give qualitatively relevant results, consistent with the announced goal of this section, namely, “*avoiding to make the satellites’ dynamics over-stable because of considering a fixed obliquity for Jupiter.*” However, the inertial terms given by Appendix B.3 are quantitatively incorrect. The corrected version of Appendix B.3 is given below. The rest of the article remains unchanged.

Appendix B.3 Inertial terms

Once an orbital and rotational solution for Jupiter is known, the computation of the inertial term \mathcal{H}_I at any time is straightforward. As explained in Appendix A, the vector Θ is the rotation velocity of our rotating reference frame \mathcal{R}' (with the z axis perpendicular to Jupiter’s equator and the x axis directed towards its equinox) measured in a nonrotating reference frame \mathcal{R} . For instance, the rotation matrix R that converts the coordinates of a vector expressed in our reference frame towards the J2000 ecliptic and equinox reference frame is

$$R = R_z(\Omega)R_x(I)R_z(-\Omega)R_z(-\psi)R_x(-\varepsilon), \quad (1)$$

where

$$R_x(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{pmatrix}, \quad R_z(\alpha) = \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2)$$

All angles Ω , I , ψ , and ε are functions of time t . If we write $(\mathbf{e}'_1, \mathbf{e}'_2, \mathbf{e}'_3)$ the unitary basis vector in \mathcal{R}' , their derivatives measured in \mathcal{R} are by definition

$$\frac{d\mathbf{e}'_1}{dt} = \Theta \times \mathbf{e}'_1; \quad \frac{d\mathbf{e}'_2}{dt} = \Theta \times \mathbf{e}'_2; \quad \frac{d\mathbf{e}'_3}{dt} = \Theta \times \mathbf{e}'_3. \quad (3)$$

The instantaneous vectors $(\mathbf{e}'_1, \mathbf{e}'_2, \mathbf{e}'_3)$ measured in \mathcal{R} are easily expressed using the matrix R ; they are, namely, the first, the second, and the third columns of the matrix. Likewise, their

time derivatives are the columns of dR/dt , which is computed using the chain rule from the time derivatives of $(\Omega, I, \psi, \varepsilon)$. In order to compute the three components of Θ , we must solve Eq. (3). To do so, we first note that, by definition, the vectors $d\mathbf{e}'_i/dt$ can be:

1. All non zero. In that case, they all lie in the same plane, which is perpendicular to Θ .
2. One is zero and the two other ones are non-zero. In that case, the two non-zero vectors are not collinear (in fact, they are perpendicular). This is the case if Θ is along one basis vector of \mathcal{R} .
3. All three vectors are zero. In that case, $\Theta = \mathbf{0}$. The rotation between the two reference frames is stationary, R is a constant matrix over time.

Based on these properties we can set up a generic procedure to compute Θ . We first compute the three cross products \mathbf{w}_{12} , \mathbf{w}_{13} , and \mathbf{w}_{23} where

$$\mathbf{w}_{ij} = \frac{d\mathbf{e}'_i}{dt} \times \frac{d\mathbf{e}'_j}{dt}. \quad (4)$$

We then select the vector \mathbf{w}_{ij} with largest modulus. If this modulus is zero, then we are in case 3 and $\Theta = \mathbf{0}$. If this modulus is not zero, we define the unitary vector

$$\mathbf{n} = \frac{\mathbf{w}_{ij}}{\|\mathbf{w}_{ij}\|}. \quad (5)$$

By definition, Θ can be written as $\Theta = \omega \mathbf{n}$, where ω is the spin rate of \mathcal{R}' with respect to \mathcal{R} .

It now remains to compute the value of ω (which can be positive or negative with these conventions). To do so, we first note that if i, j, k are in the direct order (i.e. $\mathbf{e}'_i \times \mathbf{e}'_j = \mathbf{e}'_k$), we have the following equality:

$$\frac{d\mathbf{e}'_i}{dt} \cdot \mathbf{e}'_j = \mathbf{e}'_j \cdot (\Theta \times \mathbf{e}'_i) = \Theta \cdot (\mathbf{e}'_i \times \mathbf{e}'_j) = \Theta \cdot \mathbf{e}'_k = \omega \cos \alpha, \quad (6)$$

where α is the angle between \mathbf{n} and \mathbf{e}'_k . Moreover, we have

$$\frac{d\mathbf{e}'_k}{dt} \cdot (\mathbf{n} \times \mathbf{e}'_k) = \omega \sin^2 \alpha. \quad (7)$$

Using these equalities, we can finally express ω as

$$\omega = \left(\frac{d\mathbf{e}'_i}{dt} \cdot \mathbf{e}'_j \right) (\mathbf{n} \cdot \mathbf{e}'_k) + \frac{d\mathbf{e}'_k}{dt} \cdot (\mathbf{n} \times \mathbf{e}'_k). \quad (8)$$

This formula is always valid and it avoids any division by potentially small numbers. In practice, due to computational errors, the modulus of \mathbf{w}_{ij} may occasionally be very close to zero but not zero. In that case the vector \mathbf{n} has no physical meaning and can have any orientation; yet, this is not a problem, because we will then obtain $\omega = 0$ and get anyway $\Theta = \mathbf{0}$ as required.